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FEW MAGNETOHYSDROSTATIC FORMS IN CONFOCAL PARABOLOIDAL DUCTS

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ABSTRACT

In the present paper it is proposed to study some magnetic fields with conservative Lorentz force in confocal paraboloidal coordinates, neglecting the effects of displacement current. It has been shown that self-superposable flow of an electrically conducting incompressible fluid permeated by a magnetic field with conservative Lorentz force may constitute a magnetohydrostatic configuration under certain conditions. Some such possible configurations have been attempted for the conducting fluid flowing in confocal paraboloidal ducts and pressure distributions of these configurations have also been determined.

KEYWORDS: MHD Flow and Lorentz Force

INTRODUCTION

Here we have used these techniques frequently in some important conclusions about MHD duct flow. Mittal et al (6, 7, 8, 9 and 10) have discussed some of such problems in ducts of different shapes and cross-sections.

Lorentz force plays many important roles in MHD flows. Various authors (1, 2 and 6) have represented it in various coordinate systems. It is also proposed to discuss some magnetic fields in oblate spheroidal coordinates for which Lorentz force is conservative. Some magnetic fields whose Lorentz force can be represented by the gradient of a scalar quantity have been determined.

The basic assumption that displacement current field being small compared to the prevalent electric field can be discarded, have been taken into account in equations of the problem. The magnetohydrostatic configurations can be developed when the magnetic fields with conservative Lorentz force will act upon self-superposable flow of electrically conducting fluids. It is attempted to find out the conditions of such magnetohydrostatic configurations have been studied by Mittal (6, 7), Mittal, Thapaliyal and Khan (9, 10), Mittal and Khan (8) for the ducts of different shapes.

LORENTZ FORCE

Let the Lorentz force be represented by

$$\vec{L} = \frac{-\mu}{4\pi}\left(\vec{H} \times curl\vec{H}\right) \tag{1.1}$$

Here $\vec{H}$ and μ denote the magnetic field and permeability respectively. Now suppose that ϕ be required scalar so that we may express $\vec{L}$ as

$$\vec{L} = \frac{-\mu}{4\pi} grad\phi \tag{1.2}$$

Equating the various terms of (1.1) and (1.2) in confocal paraboloidal coordinates (11), we get

$$L_u = \frac{\mu}{2\pi}\left[\frac{H_w}{(w-u)}\left\{2\sqrt{\frac{(a^2-w)(b^2-w)}{(w-v)}}\frac{\partial}{\partial w}(\sqrt{w-u}H_u) - \sqrt{\frac{(a^2-u)(b^2-u)}{(\mathrm{u}-w)}}\frac{\partial}{\partial u}(\sqrt{u-w}H_w)\right\}\right.$$

$$\left. -\frac{H_v}{(\mathrm{u}-w)}\left\{\sqrt{\frac{(a^2-w)(b^2-w)}{(u-w)}}\frac{\partial}{\partial u}(\sqrt{u-v}H_v)\right\}\right] \tag{1.3}$$

$$L_v = \frac{\mu}{2\pi}\left[\frac{H_u}{(\mathrm{u}-w)}\left\{\sqrt{\frac{(a^2-u)(b^2-u)}{(\mathrm{u}-w)}}\frac{\partial}{\partial u}(\sqrt{u-v}H_v) - \sqrt{\frac{(a^2-v)(b^2-v)}{(\mathrm{v}-w)}}\frac{\partial}{\partial v}(\sqrt{v-u}H_u)\right\}\right.$$

$$\left. -\frac{H_w}{(\mathrm{v}-w)}\left\{\sqrt{\frac{(a^2-v)(b^2-v)}{(\mathrm{v}-u)}}\frac{\partial}{\partial v}(\sqrt{v-w}H_w) - 2\sqrt{\frac{(a^2-w)(b^2-w)}{(\mathrm{w}-u)}}\frac{\partial}{\partial w}(\sqrt{w-v}H_v)\right\}\right] \tag{1.4}$$

$$L_w = \frac{\mu}{4\pi}\left[\frac{H_v}{(\mathrm{v}-w)}\left\{\sqrt{\frac{(a^2-v)(b^2-v)}{(\mathrm{v}-u)}}\frac{\partial}{\partial v}(\sqrt{v-w}H_w) - 2\sqrt{\frac{(a^2-w)(b^2-w)}{(\mathrm{w}-u)}}\frac{\partial}{\partial w}(\sqrt{w-v}H_v)\right\} - \frac{H_u}{(\mathrm{w}-u)}\left\{2\sqrt{\frac{(a^2-w)(b^2-w)}{(\mathrm{w}-v)}}\frac{\partial}{\partial w}(\sqrt{w-u}H_u)\right.\right.$$

$$\left.\left. -\sqrt{\frac{(a^2-u)(b^2-u)}{(\mathrm{u}-v)}}\frac{\partial}{\partial u}(\sqrt{u-w}H_w)\right\}\right] \tag{1.5}$$

Here (L_u, L_v, L_w) and $(H_u, \mathrm{H}_v, \mathrm{H}_w)$ are the coordinates of $\vec{L}$ and $\vec{H}$ at every point (u,v,w) in confocal paraboloidal coordinates. Also,

$$grad\phi = 2\sqrt{\frac{(a^2-u)(b^2-u)}{(v-u)(w-u)}}\frac{\partial\phi}{\partial u}\vec{i}_1 + 2\sqrt{\frac{(a^2-v)(b^2-v)}{(\mathrm{w}-v)(\mathrm{u}-v)}}\frac{\partial\phi}{\partial v}\vec{i}_2 + 2\sqrt{\frac{(a^2-w)(b^2-w)}{(\mathrm{u}-w)(\mathrm{v}-w)}}\frac{\partial\phi}{\partial w}\vec{i}_3 \tag{1.6}$$

where $\vec{i}_1, \vec{i}_2, \vec{i}_3$ are the unit vectors along u, v, w axes respectively. From equation (1.2) with the help of equations (1.3), (1.4), (1.5) and (1.6), we get,

$$\frac{\partial\phi}{\partial u} = \frac{1}{2}\sqrt{\frac{(v-u)(w-u)}{(\mathrm{a}^2-u)(b^2-u)}}L_u \tag{1.7}$$

$$\frac{\partial\phi}{\partial v} = \frac{1}{2}\sqrt{\frac{(\mathrm{w}-v)(\mathrm{u}-v)}{(\mathrm{a}^2-v)(b^2-v)}}L_v \tag{1.8}$$

$$\frac{\partial\phi}{\partial w}=\frac{1}{2}\sqrt{\frac{(u-w)(v-w)}{(a^2-w)(b^2-w)}}L_w \tag{1.9}$$

Values of $\frac{\partial\phi}{\partial u},\frac{\partial\phi}{\partial v},\frac{\partial\phi}{\partial w}$ when substituted in

$$\phi=\int\left(\frac{\partial\phi}{\partial u}du+\frac{\partial\phi}{\partial v}dv+\frac{\partial\phi}{\partial w}dw\right) \tag{1.10}$$

will give ϕ.

Also, the magnetic field $\vec{H}$ should satisfy the continuity equation $div\vec{H}=0$

$$\frac{2\sqrt{(a^2-u)(b^2-u)}}{(u-v)(w-u)}\frac{\partial}{\partial u}\sqrt{(u-v)(u-w)}H_u+\frac{2\sqrt{(a^2-v)(b^2-v)}}{(u-v)(v-w)}\frac{\partial}{\partial v}\sqrt{(v-w)(v-u)}H_v$$

$$+\frac{2\sqrt{(a^2-w)(b^2-w)}}{(v-w)(w-u)}\frac{\partial}{\partial w}\sqrt{(w-u)(w-v)}H_w=0 \tag{1.11}$$

In order to make equation (1.11) integrable we may consider the following cases

Case I- when $H_u=0$. In this case (1.11) will be satisfied by the field

(i)
$$\left.\begin{aligned}&H_u=0\\&H_v=\frac{\alpha F(u)\theta(w)}{\sqrt{(v-w)(v-u)}}\\&H_w=\frac{\beta F_1(u)\psi(v)}{\sqrt{(w-u)(w-v)}}\end{aligned}\right\} \tag{1.12}$$

where $F(u),F_1(u)$ are integrable functions of u; $\psi(v)$ and $\theta(w)$ the integrable functions of v and w respectively and α and β the constants.

For this magnetic field, it can be easily shown that Lorentz force $\vec{L}$ can be represented by the gradient of a scalar quantity ϕ given by

$$\phi=\left[\frac{\alpha^2\{\theta(w)\}^2}{(w-v)}\int\frac{F(u)F'(u)}{(v-u)}du-\frac{\beta^2\{\psi(v)\}^2}{(w-v)}\int\frac{F_1(u)F_1'(u)}{(w-u)}du+\frac{\beta^2\{F_1(u)\}^2}{(w-u)}\int\frac{\psi(v)\psi'(v)}{(w-v)}dv\right.$$

$$-\frac{2\alpha\beta F(u)F_1(u)\theta'(w)}{(w-v)}\sqrt{(a^2-w)(b^2-w)}\int\sqrt{\frac{\psi(v)dv}{(w-v)\sqrt{(a^2-v)(b^2-v)}}}$$

$$+\frac{\alpha\beta F(u)F_1(\mathrm{u})\psi'(\mathrm{v})\sqrt{(a^2-v)(b^2-v)}}{2(\mathrm{v}-u)}\int\sqrt{\frac{\theta(\mathrm{w})dw}{(\mathrm{v}-w)\sqrt{(a^2-w)(b^2-w)}}}-\frac{\alpha^2\{\mathrm{F(u)}\}^2}{(\mathrm{v}-u)}\int\frac{\theta(\mathrm{w})\theta'(\mathrm{w})}{(v-w)}dw\Bigg] \tag{1.13}$$

Now it is clear that magnetic field (12) must have Lorentz conservative force as it is represented by the gradient of a scalar quantity ϕ given by equation (3.13).

By choosing different suitable sets of values of F (u), F_1(u), $\psi(v)$ and $\theta(w)$, we get a number of magnetic fields with conservative Lorentz force. One of such field can be obtained by taking

$$\left.\begin{aligned} &F(u)=F_1(u)=\sqrt{(a^2-u)(b^2-u)}\\ &\psi(v)=\sqrt{(a^2-v)(b^2-v)}\\ &\theta(w)=\sqrt{(a^2-w)(b^2-w)}\\ &\beta=2\alpha \end{aligned}\right\} \tag{1.14}$$

The magnetic field will become

$$\left.\begin{aligned} &H_u=0\\ &H_v=\alpha\sqrt{\frac{(a^2-u)(b^2-u)(a^2-w)(b^2-w)}{(v-w)(v-u)}}\\ &H_w=2\alpha\sqrt{\frac{(a^2-u)(b^2-u)(a^2-v)(b^2-v)}{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.15}$$

for this magnetic field represented by equation (1.15), the Lorentz force can be represented by the gradient of a scalar quantity ϕ given by

$$\phi=\alpha^2\Bigg[\frac{(a^2-w)(b^2-w)}{(w-v)}\{(\frac{a^2+b^2}{2}-v)\log(v-u)-u\}-\frac{4(a^2-v)(b^2-v)}{(w-v)}\{(\frac{a^2+b^2}{2}-w)\log(\mathrm{w}-\mathrm{u})-\mathrm{u}\}$$

$$+\frac{(a^2-v)(b^2-v)(w^2-4v^2+4uv-uw)}{(w-u)(v-u)}\Bigg] \tag{1.16}$$

Similarly one more magnetic field with conservative Lorentz force is given by

$$\left.\begin{aligned} &H_u=0\\ &H_v=\frac{\gamma}{\sqrt{(v-w)(v-u)}}\\ &H_w=\frac{\delta}{\sqrt{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.17}$$

then $L_u=L_v=L_w=0$

and ϕ = constant (1.18)

Then the magnetic field (1.17) becomes force free field.

Case II- When $H_v = 0$, in this case the magnetic field with conservative Lorentz force is given by

(i)
$$\left.\begin{aligned} H_u &= \frac{\alpha_1\psi_1(v)\theta_1(w)}{\sqrt{(u-v)(u-w)}} \\ H_v &= 0 \\ H_w &= \frac{\beta_1 F_2(\mathrm{u})\psi_2(\mathrm{v})}{\sqrt{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.19}$$

and

$$\phi = \left[\frac{2\alpha_1\beta_1\psi_1(v)\psi_2(v)\theta_1'(w)\sqrt{(a^2-w)(b^2-w)}}{(w-v)} \times \int \frac{F_2(u)du}{(w-u)\sqrt{(a^2-w)(b^2-w)}} \right.$$

$$-\frac{\beta_1^2\{\psi_2(v)\}^2}{(w-v)}\int\frac{F_2(u)F_2'(u)du}{(w-u)} + \frac{\alpha_1^2\{\theta_1(\mathrm{w})\}^2}{(w-v)}\int\frac{\psi_1(u)\psi_1'(u)dv}{(\mathrm{u}-v)}$$

$$+\frac{\beta_1^2\{\mathrm{F}_2(\mathrm{u})\}^2}{(w-u)}\int\frac{\psi_2(\mathrm{v})\psi_2'(\mathrm{v})dv}{(w-v)} + \frac{\alpha_1^2\{\psi_1(\mathrm{v})\}^2}{(\mathrm{u}-v)}\int\frac{\theta_1(\mathrm{w})\theta_1'(\mathrm{w})dw}{(\mathrm{u}-w)}$$

$$\left. -\frac{\alpha_1\beta_1\psi_1(v)\psi_2(v)F_2'(u)\sqrt{(a^2-u)(b^2-u)}}{2(u-v)} \times \int \frac{\theta_1(w)dw}{(\mathrm{u}-\mathrm{w})\sqrt{(a^2-w)(b^2-w)}} \right] \tag{1.20}$$

(ii)
$$\left.\begin{aligned} H_u &= \alpha_1\sqrt{\frac{(a^2-v)(b^2-v)(a^2-w)(b^2-w)}{(u-v)(u-w)}} \\ H_v &= 0 \\ H_w &= 2\alpha_1\sqrt{\frac{(a^2-u)(b^2-u)(a^2-v)(b^2-v)}{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.21}$$

and

$$\phi = \alpha_1^2\left[\frac{4(a^2-u)(b^2-u)}{(w-u)}\{(\frac{a^2+b^2}{2}-w)\log(w-v)-v\} + \frac{(a^2-v)(b^2-v)(4u^2+w^2-4uv-vw)}{(w-v)(u-v)} \right.$$

$$\left. +\frac{(a^2-w)(b^2-w)}{(w-u)}\{(\frac{a^2+b^2}{2}-u)\log(\mathrm{u}-v)-v\} \right] \tag{1.22}$$

(iii)
$$\left.\begin{aligned} H_u &= \frac{\gamma_1}{\sqrt{(u-v)(u-w)}} \\ H_v &= 0 \\ H_w &= \frac{\delta_1}{\sqrt{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.23}$$

and ϕ =constant (1.24)

i.e. the magnetic field is force free type

Case III-when $H_w = 0$, some magnetic field with conservative Lorentz force are given by

(i)
$$\left.\begin{aligned} H_u &= \frac{\alpha_2\psi_3(v)\theta_2(w)}{\sqrt{(u-v)(u-w)}} \\ H_v &= \frac{\beta_2 F_3(\mathrm{u})\theta_3(w)}{\sqrt{(\mathrm{v}-w)(\mathrm{v}-u)}} \\ H_w &= 0 \end{aligned}\right\} \tag{1.25}$$

and

$$\phi = \Bigg[\frac{\beta_2^2\{\theta_3(w)\}^2}{(v-w)} \int \frac{F_3(u)\,\mathrm{F}_3'(u)}{(v-u)} du$$

$$-\frac{\alpha_2\beta_2\psi_3'(v)\theta_2(w)\theta_3(w)\sqrt{(a^2-v)(b^2-v)}}{(v-w)} \times \int \frac{F_3(u)\,\mathrm{du}}{(v-u)\sqrt{(a^2-w)(b^2-w)}}$$

$$+\frac{\alpha_2\beta_2 F_3'(\mathrm{u})\theta_2(w)\theta_3(w)\sqrt{(a^2-u)(b^2-u)}}{(\mathrm{u}-w)} \times \int \frac{\psi_3(\mathrm{v})\,\mathrm{dv}}{(\mathrm{u}-v)\sqrt{(a^2-v)(b^2-v)}}$$

$$-\frac{\alpha_2^2\{\theta_2(w)\}^2}{(\mathrm{u}-w)} \int \frac{\psi_3(\mathrm{v})\psi_3'(\mathrm{v})}{(\mathrm{u}-v)} dv + \frac{\beta_2^2\{\mathrm{F}_3(\mathrm{u})\}^2}{(v-u)} \int \frac{\theta_3(\mathrm{w})\theta_3'(\mathrm{w})}{(\mathrm{w}-v)} dw - \frac{\alpha_2^2\{\psi_3(\mathrm{v})\}^2}{(v-u)} \int \frac{\theta_2(\mathrm{w})\theta_2'(\mathrm{w})}{(\mathrm{w}-u)} dw \tag{1.26}$$

(ii)
$$\left.\begin{aligned} H_u &= \alpha_2\sqrt{\frac{(a^2-v)(b^2-v)(a^2-w)(b^2-w)}{(u-v)(u-w)}} \\ H_v &= \alpha_2\sqrt{\frac{(a^2-u)(b^2-u)(a^2-w)(b^2-w)}{(\mathrm{v}-w)(\mathrm{v}-u)}} \\ H_w &= 0 \end{aligned}\right\} \tag{1.27}$$

Then $\phi = \alpha_2^2 \left[\frac{(a^2-w)(b^2-w)(v-u)(u+v-w)}{(u-w)(v-w)} + \frac{(a^2-u)(b^2-u)}{(v-u)}\{w-(\frac{a^2+b^2}{2}-v)\log(w-v)\} \right.$

$$\left. -\frac{(a^2-v)(b^2-v)}{(v-u)}\{w-(\frac{a^2+b^2}{2}-u)\log(w-u)\} \right] \tag{1.28}$$

(iii)

$$\left. \begin{aligned} H_u &= \frac{\gamma_2}{\sqrt{(u-v)(u-w)}} \\ H_v &= \frac{\delta_2}{\sqrt{(v-u)(v-w)}} \\ H_w &= 0 \end{aligned} \right\} \tag{1.29}$$

for this field ϕ = constant (1.30)

In the above discussion all the nine magnetic fields in confocal paraboloidal coordinates given by (1.12), (1.15), (1.17), (1.19), (1.21), (1.23), (1.25), (1.27) and (1.29) have conservative Lorentz values and one of these (1.12), (1.15), (1.19), (1.21), (1.25), (1.27) are pressure balanced fields and it is a well-known fact that in such fields lines of force and current are situated in the surface ϕ = constant. The other fields (1.17), (1.23) and (1.29) are force free magnetic fields.

SOME MAGNETOHYDROSTATIC CONFIGURATIONS

If $\vec{q}$ denotes the fluid velocity of an electrically conducting incompressible flud permeated by a magnetic field $\vec{H}$ and ρ the density, the steady state consists of all solutions of the equation given by (4).

$$\rho curl(\vec{q})\times\vec{q} - \frac{\mu}{4\pi}(curl\vec{H})\times\vec{H} = -gradp - \rho grad(\frac{1}{2}\mathrm{q}^2) + \vec{\mathrm{F}} \tag{1.31}$$

$$curl(\vec{q}\times\vec{H}) = 0 \tag{1.32}$$

Here the body force $\vec{F}$ will contain no frictional force.

Let q_u, q_v and q_w be the components of $\vec{q}$ at any point (u, v, w) in confocal paraboloidal coordinates.

- $$\left. \begin{aligned} q_u &= 0 \\ q_v &= \frac{AUW}{\sqrt{(v-w)(v-u)}} \\ q_w &= \frac{BU_1V}{\sqrt{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned} \right\} \tag{1.33}$$

- $$\left.\begin{aligned} q_u &= 0 \\ q_v &= A\sqrt{\frac{(a^2-u)(b^2-u)(a^2-w)(b^2-w)}{(v-w)(v-u)}} \\ q_w &= 2A\sqrt{\frac{(a^2-u)(b^2-u)(a^2-v)(b^2-v)}{(w-u)(\mathrm{w}-\mathrm{v})}} \end{aligned}\right\} \tag{1.34}$$

- $$\left.\begin{aligned} q_u &= 0 \\ q_v &= \frac{C_1}{\sqrt{(v-w)(v-u)}} \\ q_w &= \frac{D_1}{\sqrt{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.35}$$

- $$\left.\begin{aligned} q_u &= \frac{A_1V_1W_1}{\sqrt{(u-v)(u-w)}} \\ q_v &= 0 \\ q_w &= \frac{B_1U_2V_2}{\sqrt{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.36}$$

- $$\left.\begin{aligned} q_u &= A_1\sqrt{\frac{(a^2-v)(b^2-v)(a^2-w)(b^2-w)}{(u-v)(u-w)}} \\ q_v &= 0 \\ q_w &= 2A_1\sqrt{\frac{(a^2-u)(b^2-u)(a^2-v)(b^2-v)}{(\mathrm{w}-u)(\mathrm{w}-v)}} \end{aligned}\right\} \tag{1.37}$$

- $$\left.\begin{aligned} q_u &= \frac{C_2}{\sqrt{(u-v)(u-w)}} \\ q_v &= 0 \\ q_w &= \frac{D_2}{\sqrt{(w-u)(w-v)}} \end{aligned}\right\} \tag{1.38}$$

- $$\left.\begin{aligned} q_u &= \frac{A_2V_3W_2}{\sqrt{(u-v)(u-w)}} \\ q_v &= \frac{B_2U_3W_3}{\sqrt{(\mathrm{v}-w)(\mathrm{v}-u)}} \\ q_w &= 0 \end{aligned}\right\} \tag{1.39}$$

- $$\left.\begin{aligned} q_u &= A_2\sqrt{\frac{(a^2-v)(b^2-v)(a^2-w)(b^2-w)}{(u-v)(u-w)}} \\ q_v &= A_2\sqrt{\frac{(a^2-u)(b^2-u)(a^2-w)(b^2-w)}{(\mathrm{v}-w)(\mathrm{v}-u)}} \\ q_w &= 0 \end{aligned}\right\} \tag{1.40}$$

- $$\left.\begin{aligned} q_u &= \frac{C_2}{\sqrt{(u-v)(u-w)}} \\ q_v &= \frac{D_2}{\sqrt{(v-w)(v-u)}} \\ q_w &= 0 \end{aligned}\right\} \tag{1.41}$$

Where U, U_1, U_2, U_3 are the integrable functions of u; V, V_1, V_2, V_3 are functions of v and W, W_1, W_2, W_3 are the functions of w. Also A, A_1, A_2, B, B_1, B_2, C, C_1, C_2, D, D_1, and D_2 are constants which may be determined by boundary conditions.

Now, let us consider the flow of an electrically conducting incompressible fluid with velocity $\vec{q}$ [given by equation (1.33)] acted upon by the magnetic field given by equation (1.12), if

$$\left.\begin{aligned} F(u) &= U(u) \\ F_1(u) &= U_1(u) \\ \theta(w) &= W(w) \\ \psi(v) &= V(v) \\ \alpha &= \pm\sqrt{\frac{\mu}{4\pi}}A \\ \beta &= \pm\sqrt{\frac{\mu}{4\pi}}B \end{aligned}\right\} \tag{1.42}$$

Then the flow (1.33) and magnetic field (1.12) will satisfy the equation (1.32). Also we have

$$\vec{q} = \pm\sqrt{\frac{\mu}{4\pi\rho}}\vec{H} \tag{1.43}$$

Walen (13, 14) has shown that equation (1.42) is always a solution of equation (1.32) if $\vec{F} = 0$ (no external forces) and if (13)

$$P + \frac{1}{2}\rho q^2 = \cos t. \tag{1.44}$$

Thus the flow of an electrically conducting incompressible fluid with velocity given by equation (1.33) acted upon by the magnetic field given by equation (1.12) constitute a magneto-hydrostatic configuration if there are no external forces, then equation (1.42) is satisfied and pressure distribution is given by

$$P = const. - \frac{1}{(v-w)}\left[\frac{\lambda_1 U^2 W^2}{(v-u)} - \frac{\lambda_2 U_1{}^2 V^2}{(w-u)}\right] \tag{1.45}$$

CONCLUSIONS

Magnetohydrostatic configurations thus found may be either pressure balanced type or in very special cases force free type (5). In all these, no secondary flow will be possible because the magnetic forces would just balance the centrifugal effects (12).

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